PHYSICAL REVIEW E VOLUME 62, NUMBER 4 OCTOBER 2000

Relaxation processes in administered-rate pricing

Raymond J. Hawkins[1] and Michael R. Arnold[2]
[1]*Bear, Stearns Securities Corporation, 220 Bush Street, Suite 845, San Francisco, California 94104*
[2]*Deloitte & Touche, Capital Markets Group, 50 Fremont Street, San Francisco, California 94105*

(Received 18 April 2000)

We show how the theory of anelasticity unifies the observed dynamics and proposed models of administered-rate products. This theory yields a straightforward approach to rate model construction that we illustrate by simulating the observed relaxation dynamics of two administered rate products. We also demonstrate how the use of this formalism leads to a natural definition of market friction.

PACS number(s): 89.90.+n, 62.40.+i, 81.40.Jj, 83.50.By

I. INTRODUCTION

Administered-rate products represent a substantial fraction of the liabilities of banks and savings and loans[1] and the ability to describe the response of these products to changes in market rates is of critical importance for interest-rate risk management.[2] Unlike market rates (e.g., U.S. Treasury bonds) that are set in an auction environment or contractual rates that respond instantaneously to market rates in a prescribed manner (e.g., home mortgages), administered rates (e.g., interest rates on checking and savings accounts) are set by a committee seeking an equilibrium rate in response to changes in market rates. Factors that bear on this equilibrium pricing include expected future market rates, competitor pricing responses, and depositor's short-term and long-term balance elasticities (i.e., propensity to change balance levels in response to rate changes); all of which are known with limited certainty. Balance elasticities are exceedingly difficult to estimate with any reliability because banks usually do not preserve much historical data and do not usually employ the resources needed to evaluate the data that does exist. Consequently, these committees indicate a certain amount of inertia.

The general proclivity of pricing committees to leave well enough alone has been found in a variety of empirical studies [3–9] that have established that administered rates are driven by market rates and that the response of an administered rate to the market rate is not instantaneous. This behavior has been modeled by a number of investigators [1,2,8–13] who have settled largely on the use of partial adjustment models[3] to describe the noninstantaneous response of administered rates to changes in market rates. While these models often adequately describe the observed administered-rate behavior, they largely lack a theoretical basis with which to interpret the resulting parameters and with which to link rate policy to the rate model. In this paper we show that the formal assumptions upon which previous treatments of administered-rate dynamics are based are identical to the assumptions underlying the formal treatment of a variety of relaxation processes in condensed-matter physics including magnetic, dielectric, and anelastic relaxations [15]. All these physical phenomena involve time-dependent relaxations toward newly established equilibria that follow from a change in a driving force and can be described in terms of linear-response theory. Since these physical phenomena share a common mathematical description of relaxation/response and since these physical phenomena and administered-rate dynamics share a variety of underlying assumptions, we make the *ansatz* that these phenomena all share a common mathematical description. Given this we can move beyond an *ad hoc* treatment of administered-rate dynamics and employ the phenomenological models that have been developed to model these physical relaxation processes to model the dynamics of administered-rate deposits.

The theory of anelasticity provides a useful framework with which to develop our treatment of administered-rate dynamics because of the similarity between some of the equations that have appeared in the literature on administered rates and the scalar representation of anelasticity. We show in Sec. II how the theory of anelasticity can be used to develop a hierarchy of continuous-time models of the administered-rate response function that include, as a subset, the partial adjustment models that have appeared in the literature. We find that the continuous-time models that follow from this approach lend themselves to an easy mapping between certain aspects of rate policy and of model structure.

---

[1]Mays [1] notes that nonmaturity deposits—a class of administered-rate products—comprise "42% of total bank liabilities and over 25% of savings and loan (S&L) liabilities as of December 1995."

[2]As pointed out by Mays [1] and O'Brien *et al.* [2] the present value PV of administered-rate nonmaturity deposits can be estimated using the equation

$$PV = D_0 - \sum_{t=1}^{\infty} \frac{D_{t-1}(r_t^{(m)} - r_t^{(d)} - c)}{(1+r_t^{(m)})^t}, \tag{1.1}$$

where D_t is the deposit balance at time t, $r_t^{(m)}$ is the market rate, $r_t^{(d)}$ is the administered deposit rate, and c denotes the noninterest charges associated with maintaining the account. Clearly there is interest rate sensitivity in the present value: explicitly in the appearance of the market rate in Eq. (1) and implicitly in both the sensitivity of the balances and the administered deposit rate to changes in the market rate.

[3]See, for example, Chap. 9 of Kennedy [14] for a discussion of partial adjustment models in econometrics.

We illustrate the utility of this approach in Sec. III by modeling the rate-response behavior for two administered-rate products: money market accounts and time deposits. In Sec. IV we explore the notion of dissipation embodied in these relaxation dynamics to develop a formal notion of market friction.

II. RATE DYNAMICS

A. Assumptions and econometric models

Fundamental to essentially all descriptions of administered-rate product rates is the notion that there exists an equilibrium relationship between the product rate $\tilde{r}^{(p)}$ and the market rate $r^{(m)}$ that is of the form

$$\tilde{r}^{(p)}=c+Jr^{(m)}, \tag{2.1}$$

where the tilde indicates equilibrium. The constant c is often taken to denote the costs to the bank of servicing the product. The proportionality factor J has, in the case of nonmaturity deposits, been interpreted as the fraction of deposited monies that Federal Reserve requirements allow to be lent [2,16,17]. The intuition behind this equilibrium relationship, as pointed out by O'Brien *et al.* [2], is "that if a bank offered to pay the market rate on an account the bank would lose money for two reasons. First, due to Federal Reserve requirements, the bank cannot reinvest and earn interest on all of a deposit, but only on the fraction J of the account; where J equals one minus the marginal reserve requirement. But even the deposit rate $Jr^{(m)}$ loses money for the bank since the bank must cover its costs of servicing the account." Thus, c is added to $Jr^{(m)}$ and we obtain Eq. (2.1). While the interpretation of J in terms of a Federal Reserve requirement clearly breaks down for products without such a requirement, Eq. (2.1) remains, nevertheless, a basic assumption of equilibrium behavior for most administered-rate products [1,2,8–13].

The rate relationship in Eq. (2.1) is characterized by three features: (i) a unique equilibrium product rate for each level of the market rate, (ii) instantaneous achievement of the equilibrium response, and (iii) linearity of the response. We note in passing that the equilibrium rate is completely recoverable.

The empirical dynamics of administered rates, however, demonstrate that the equilibrium response is not achieved instantaneously and a lagged response is observed [3–9]. To incorporate this observed lag into the relationship between the product rate and the market rate, previous research has augmented Eq. (2.1) with an *ad hoc* "partial adjustment" model of the form

$$\Delta r_n^{(p)}=\sum_{i=0}^{N}\,[a_i r_{n-i}^{(m)}+b_i r_{n-i+1}^{(p)}], \tag{2.2}$$

where we have introduced the time-dependent notation $r_n^{(p)} \equiv r^{(p)}(t_n)$. This functional form is the basis of most of the econometric studies that have appeared in the literature to date [1,2,8–13]. While some researchers have posited that the coefficients in Eq. (2.2) depend on the direction of the change in the market rate, O'Brien [13] has recently pointed out, however, that such an assumption is not consistent with the assumed equilibrium relationship given in Eq. (2.1).

B. Anelastic rate dynamics

The theory of anelasticity is a generalization of the theory of ideal elasticity that allows for time dependence in the response of a material to an applied stress. Like previous treatments of administered-rate dynamics it assumes the existence of a unique equilibrium relationship between stress and strain known as Hooke's law. The equilibrium relationship given in Eq. (2.1) is, in fact, identical to the scalar version of Hooke's law of ideal elasticity $\epsilon=J\sigma$ with the product rate playing the role[4] of strain ϵ and the market rate playing the role of stress σ.[5] In this context the constant c can be interpreted as the contribution to the product rate driven by a different "stress": the cost to the bank of maintaining the account.[6]

The differential dynamics of anelasticity are not obtained through a direct application of lagged variables via Eq. (2.2) but, rather, by noting that the assumption of linearity implies a general market-product rate relationship of the form

$$\left(f_0+f_1\frac{d}{dt}+f_2\frac{d^2}{dt^2}+\cdots\right)r^{(p)} =\left(g_0+g_1\frac{d}{dt}+g_2\frac{d^2}{dt^2}\cdots\right)r^{(m)}. \tag{2.3}$$

While the econometric application of this equation, like Eq. (2.2), requires an analysis of the number of terms needed to describe the observed dynamics, the use of Eq. (2.3) enables an economic interpretation of these terms and the coefficients. In practice a wide range of relaxation dynamics have been found to be described well by the comparatively simple differential relationship[7]

$$\frac{dr^{(p)}}{dt}+\eta(r^{(p)}-c)=J_U\frac{dr^{(m)}}{dt}+\eta J_R r^{(m)}, \tag{2.4}$$

where η denotes the rate at which the product rate relaxes to the equilibrium level, J_U denotes that fraction of the response that occurs instantaneously, and J_R denotes the ulti-

---

[4]While the notions of strain, stress, and force are also used as metaphors in economics and finance, our use of these metaphors used here is to motivate the comparison of notation.

[5]A more complete correspondence with scalar elasticity can be achieved by positing that market rates change in response to market stress $\sigma^{(m)}$ induced by market forces. To the extent that market rates respond to market forces in an essentially instantaneous manner, we can write $r^{(m)}=(K/J)\sigma^{(m)}$ from which it follows that $\tilde{r}^{(p)}=c+K\sigma^{(m)}$: Hooke's law relating the product rate to the market stress via the compliance K.

[6]We thank Leif Wennerberg for this observation.

[7]Indeed, this relationship is so ubiquitous that the resulting anelastic system is referred to as the "standard anelastic solid" [18]. Higher-order differential equations can be used to treat more complex relaxation processes. Nowick and Berry [18] show, however, that these higher-order differential processes can be represented as a linear combination of the standard anelastic solid. The interest rate dynamics discussed in this paper are described quite adequately with the standard anelastic differential equation given above.

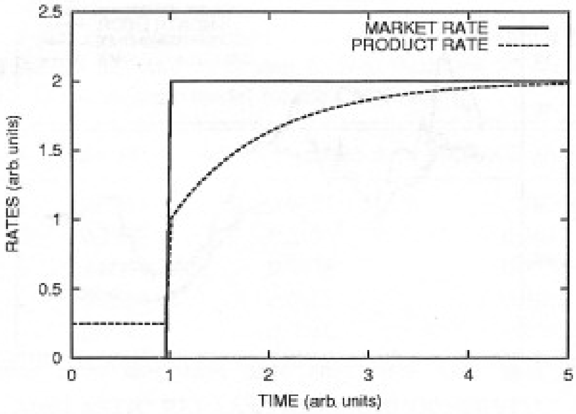

FIG. 1. The response of the product rate to a step change in the market rate given by Eq. (2.4).

mate extent of the response function $[=J(t=\infty)]$. The change in the product rate with respect to time is, in this case, a function of the current product rate, the current market rate, and the change in the market rate with respect to time.

Some intuition for the interpretation of this differential product-market rate relationship can be obtained for the case of a simple market rate shock. Given a sudden change in the market rate, that is subsequently held constant at $r^{(m)}$, and the equilibrium relationship given by Eq. (2.1), Eq. (2.4) can be integrated to yield the time-dependent product rate

$$r^{(p)}(t)=c+(J_U+\delta J[1-e^{-\eta t}])r^{(m)}, \tag{2.5}$$

where $\delta J\equiv[J_R-J_U]$, whence

$$J(t)=J_U+\delta J[1-e^{-\eta t}]; \tag{2.6}$$

illustrating the decomposition of the response J into an instantaneous contribution J_U and a time-dependent portion proportional to δJ mentioned above. The product-rate response to this step change in the market rate is illustrated in Fig. 1 where we show the response of the product rate to a step change in the market rate with $c=0.25$, $J_U=0.375$, $\delta J=0.5$, and $\eta=1.0$. The product rate tracks the market rate instantaneously over a range defined by J_U; in this case to 1.0. The product rate then relaxes to equilibrium with the market rate (in this example $\bar{r}^{(p)}=0.25+0.875r^{(m)}$). Varying J_U and J_R (or, equivalently δJ) one can span the range of responses from being completely instantaneous, $J_U=J_R>0$, to being completely time dependent, $J_U=0$.

C. Anelastic partial adjustment models

A variety of partial adjustment models can be developed for the differential relationship given above by discretizing[8] the integral form of $dr^{(p)}/dt$ given in Eqs. (2.4):

$$r^{(p)}(t)=r_n^{(p)}+\int_{t_n}^{t} f[r^{(p)}(\tau),r^{(m)}(\tau),c]d\tau, \tag{2.7}$$

[8]See, for example, Chap. 1 of Koonin [19] or Section 16.7 of Press *et al.* [20].

which is known to yield

$$r_{n+1}^{(p)}=r_n^{(p)}+h\sum_{i=0}^{N}\beta_i f_{n+1-i}, \tag{2.8}$$

where f represents all terms in Eq. (2.4) except for $dr^{(p)}/dt$ and h is the time step. This expression is essentially identical to the partial adjustment formula given in Eq. (2.2) above. Using standard discretization techniques one can derive several partial adjustment models from Eq. (2.4) appropriate for more complex behaviors of the driving market rate. We now explore some common discretizations of our standard anelastic system that resemble closely the partial adjustment formulas that have appeared in the literature.

Rewriting Eq. (2.4) in terms of the variable $y\equiv r^{(p)}-J_U r^{(m)}$, we begin with the forward-difference Euler equation $y_{n+1}=y_n+f_n$ from which it follows that

$$r_{n+1}^{(p)}=\eta c+r_n^{(p)}+J_U(r_{n+1}^{(m)}-r_n^{(m)})+\eta(J_R r_n^{(m)}-r_n^{(p)}). \tag{2.9}$$

We note that when $\eta J_R=J_U$, this relationship becomes

$$r_{n+1}^{(p)}=\eta c+(1-\eta)r_n^{(p)}+J_U r_{n+1}^{(m)}, \tag{2.10}$$

which is identical in structure to the simplest partial adjustment model discussed by Mays [1].

While the Euler discretization of Eq. (2.4) yields a popular partial adjustment expression, partial adjustment models with more temporal lags do exist in the literature on administered-rate products. More temporal lags can be introduced in two different ways. First, if the deliberations of the pricing committee are known to correspond to Eq. (2.4) (i.e., the product rate is based on considerations of the level and change of the rates) then partial adjustment models can be obtained using different discretization techniques. Alternatively, greater temporal lags follow naturally if the deliberations of the committee include the change in the slope of the market and product rates as a function of time. We examine each in turn.

Partial adjustment models, based on Eq. (2.4), with greater temporal lags, can be obtained through higher-order discretizations such as the Adams-Bashford methods. The Adams-Bashford two-step method is given by $y_{n+1}=y_n+[\frac{3}{2}f_n-\frac{1}{2}f_{n-1}]$. Applying this to Eq. (2.4) yields

$$r_{n+1}^{(p)}=\eta c+r_n^{(p)}+J_U(r_{n+1}^{(m)}-r_n^{(m)})+\tfrac{3}{2}\eta(J_R r_n^{(m)}-r_n^{(p)})-\tfrac{1}{2}\eta(J_R r_{n-1}^{(m)}-r_{n-1}^{(p)}), \tag{2.11}$$

which shares many structural aspects with the partial adjustment model developed by the Office of Thrift Supervision (OTS) [11]. Further temporal lags can be included by applying the Adams-Bashford three-step method $y_{n+1}=y_n+\frac{1}{12}[23f_n-16f_{n-1}+5f_{n-2}]$ to Eq. (2.4):

$$r_{n+1}^{(p)}=\eta c+r_n^{(p)}+J_U(r_{n+1}^{(m)}-r_n^{(m)})+\tfrac{23}{12}\eta(J_R r_n^{(m)}-r_n^{(p)})-\tfrac{16}{12}\eta(J_R r_{n-1}^{(m)}-r_{n-1}^{(p)})+\tfrac{5}{12}\eta(J_R r_{n-2}^{(m)}-r_{n-2}^{(p)}). \tag{2.12}$$

If the product rate is thought also to be a function of the change in the slope of the market and product rates as a function of time, it is likely that the dynamics are better represented by the relationship involving two relaxations

$$\frac{d^2r^{(p)}}{dt^2}+(\eta^{(1)}+\eta^{(2)})\frac{dr^{(p)}}{dt}+\eta^{(1)}\eta^{(2)}(r^{(p)}-c)$$
$$=J_U\frac{d^2r^{(m)}}{dt^2}+[\delta J^{(1)}\eta^{(1)}+\delta J^{(2)}\eta^{(2)}+(\eta^{(1)}+\eta^{(2)})J_U]$$
$$\times\frac{dr^{(m)}}{dt}+\eta^{(1)}\eta^{(2)}(\delta J^{(1)}+\delta J^{(2)}+J_U)r^{(m)}, \qquad (2.13)$$

which is simply Eq. (2.3) with up to second-order derivatives included and where $\eta^{(i)}$ and $\delta J^{(i)}$ correspond to the ith relaxation. While somewhat more formidable than Eq. (2.4), given this choice of terms and coefficients, the associated response function is known [18] to be a simple sum of the responses resulting in a generalization of Eq. (2.6) to

$$J(t)=J_U+\sum_{i=1}^{2}\delta J^{(i)}[1-e^{-\eta^{(i)}t}], \qquad (2.14)$$

where we see that the response function now contains two relaxation response terms in addition to the instantaneous response.

The partial adjustment model that follows from an Euler discretization of Eq. (2.13) yields, via the second derivatives, a function of the form

$$r_{n+1}^{(p)}=\eta c+r_n^{(p)}+h(r_n^{(p)},r_{n-1}^{(p)},r_{n+1}^{(m)},r_n^{(m)},r_{n-1}^{(m)}). \qquad (2.15)$$

While this expression contains the same number of temporal lags as Eq. (2.11) and has a structure similar to that of the OTS model [11], it has two more degrees of freedom than Eq. (2.11) due to the more complex relaxation process.

The anelastic partial adjustment models differ from the partial adjustment models that have appeared in the literature in two important ways. First, the number of free parameters is determined by the nature of the differential rate relationship, while the number of terms is determined independently by the nature of the discretization of the differential rate relationship. Second, the decoupling of the number of terms from the number of free parameters in the partial adjustment model allows the partial adjustment model to better reflect the nature of how the pricing committees adjust rates in response to market rates. While the differential rate relationship provides a phenomenological representation of the observed lag in the product-rate adjustment, the discrete form of the model can be expressed so as to reflect the number of previous time periods included in the deliberations of the pricing committee. Thus, the anelastic approach provides a convenient way to avoid overspecification of the relationship between the product and market rates while simultaneously providing the flexibility needed to properly represent the data used in rate policy decisions.

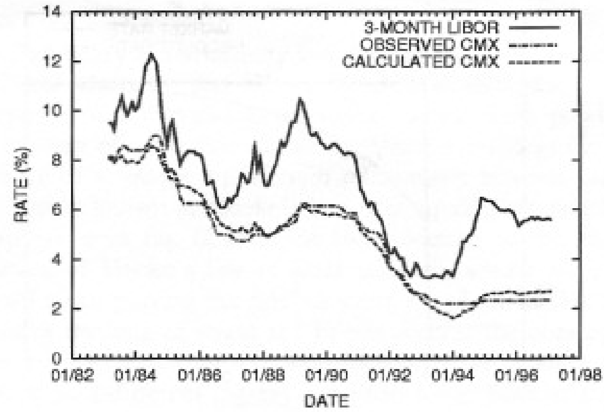

FIG. 2. The market rate (three-month LIBOR), observed CMX rate, and calculated CMX rate as a function of time.

D. Simulation using Boltzmann superposition

Having demonstrated that the functional form of the partial adjustment models that have appeared in the literature can be recovered from the anelastic formalism using standard discretization techniques, we note in passing that there is a far simpler approach to the modeling of these rate dynamics that may be of use in future work. As pointed out by Nowick and Berry [18] manipulation (and discretization) of the differential product-market rate equations becomes increasingly complex as the order of these equations increase. An appropriately chosen manner of increasing the order of the differential equation [employed, for example, in Eq. (2.13)] allows us to write down the response function [cf. Eqs. (2.6) and (2.14)] directly as

$$J(t)=J_U+\sum_{i=1}^{N}\delta J^{(i)}[1-e^{-\eta^{(i)}t}], \qquad (2.16)$$

where N represents both the order of the differential terms included in Eq. (2.3) and the number of relaxation terms. Then, as a consequence of the linearity of the system—also known as the Boltzmann superposition principle—we can, given a history of market rate changes $r_i^{(m)}$ applied at successively increasing times $\tau_1,\tau_2,\ldots,\tau_M$, write the product rate as

$$r^{(p)}(t)-c=\sum_{i=1}^{M}r_i^{(m)}J(t-\tau_i). \qquad (2.17)$$

These simple relations provide a straightforward calculation of the product rate in response to changes in the market rate with a response function that is specified easily in terms of two types of coefficients: a relaxation rate and the fraction of the response corresponding to that rate.

While by no means an exhaustive collection of the models that can be generated from Eqs. (2.1) and (2.3), Eqs. (2.9), (2.10), (2.11), (2.12), and (2.16) illustrate, nevertheless, the rich variety of partial adjustment models that follow from a single linear differential relationship between the product and market rates that, in turn, follows from an anelastic interpretation of the relationship between the product and market rates. We now apply the anelastic approach to the description of the observed dynamics of administered rates.

TABLE I. Fitted results for the (i) Euler forward difference (Euler FD), (ii) Adams-Bashford two-step (Adams-Bashford 2), and (iii) Adams-Bashford three-step (Adams-Bashford 3) discretized forms of the anelastic model for the CMX rate.

	Euler FD	Adams-Bashford 2	Adams-Bashford 3
c	−2.8544	−2.7653	−2.7006
J_U	0.2933	0.2690	0.2633
J_R	1.0060	0.9938	0.9854
η	0.0430	0.0452	0.0464
R^2	0.9741	0.9746	0.9745

III. ANELASTIC RELAXATIONS AND OBSERVED PRODUCT-RATE DYNAMICS

We have used the anelastic approach described above to model the dynamics of the Cash Maximizer™ (CMX) interest rates [21] and of the rates for retail certificates of deposit (CD's). The Cash Maximizer™ account is a money market deposit account that requires a minimum of USD 2500 to open and to avoid service charges.[9] Unlike the CMX product that has no defined maturity, the retail CD, used in this study, has a three-month maturity.[10] The CD that we shall analyze below has a face value of USD 2500. The CD and CMX rates are set by committee.

The market rate driving changes in the CMX rates is taken to be that rate most closely matching a matched rate along the bank's cost-of-funds curve. Since the marginal cost of funds for Bank of America during this period is best reflected by the London Interbank Offer Rate (LIBOR) for short maturities, we have taken the three-month LIBOR to be the market rate.

The month-end CMX rates are shown together with the three-month LIBOR for the period March 1983–February 1997 in Fig. 2. Comparing these rates we see that the CMX rate is always less than the three-month LIBOR and that the CMX rate roughly tracks the movements of the three-month LIBOR in a largely attenuated and somewhat lagged manner. We fit Eqs. (2.9), (2.11), and (2.12) to these data using the generalized reduced gradient (GRG2) nonlinear optimization solver in Microsoft Excel™. The coefficients resulting from these fits are given in Table I together with the coefficient of determination $R^2 = 1 - \mathrm{SSE/SST}$, where SSE is the error sum of squares and SST is the total sum of squares. The fit of Eq. (2.9) is shown together with the CMX rates and the three-month LIBOR rates in Fig. 2: the other fits are not shown, as differences among them cannot be resolved by eye on this scale.

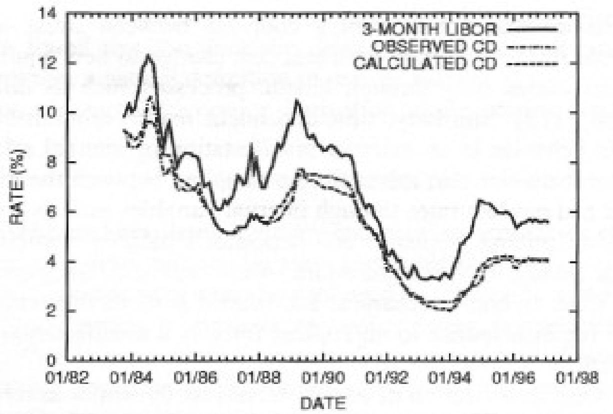

FIG. 3. The market rate (three-month LIBOR), observed CD rate, and calculated CD rate as a function of time.

An identical analysis was performed on the month-end CD rates shown, together with the three-month LIBOR for the period October 1983–February 1997, in Fig. 3. The results of the fitting described above are shown in Table II. The fit of Eq. (2.9) is shown together with the CD rates and three-month LIBOR in Fig. 3. As in the case of Fig. 2, differences among the various fits cannot be meaningfully resolved by eye on this scale.

We see in Figs. 2 and 3 that modeling changes in these product (CMX, CD) rates, as an anelastic response to a market rate, changes results in an expression [e.g., Eq. (2.9)] that can track the observed product rate quite well. This is remarkable given that these two products have quite different maturity assumptions: the CD has a fixed maturity while the CMX product has no maturity. These results, together with the observation made above that economic assumptions concerning the relationship between the market and product rates are the same as those of an anelastic process, provide compelling evidence that these product rates respond to market rates as if via anelastic relaxations.

IV. RELAXATIONS AND MARKET FRICTION

In a mechanical system the time-dependent stress-strain behavior is "an external manifestation of internal relaxation

TABLE II. Fitted results for the (i) Euler forward difference (Euler FD), (ii) Adams-Bashford two-step (Adams-Bashford 2), and (iii) Adams-Bashford three-step (Adams-Bashford 3) discretized forms of the anelastic model for the CD rate.

	Euler FD	Adams-Bashford 2	Adams-Bashford 3
c	−1.1967	−1.2097	−1.3271
J_U	0.3255	0.2932	0.3331
J_R	0.9289	0.9307	0.9493
η	0.1275	0.1332	0.1228
R^2	0.9839	0.9840	0.9840

[9]In July 1986, Bank of America introduced USD 25 000 and USD 100 000 tiers to this account. While the introduction of these tiers did introduce some additional pricing constraints (e.g., higher-minimum tiers have rates greater than or equal to lower-minimum tiers) the dynamics of each tier is otherwise considered to be independent of the other tiers.

[10]Retail customers of Bank of America could, during the period to be analyzed, select almost any maturity less than 7 years. Most customers chose the conventional maturities of 3 months, 6 months, or annual increments out to 7 years.

behavior that arises from a coupling between stress and strain through internal variables that change to new equilibrium values only through kinetic processes such as diffusion'' [18]. Similarly, time-dependent market-administered rate behavior is an external manifestation of internal relaxation behavior that arises from a coupling between the market and product rates through internal variables such as competitor pricing responses and depositor's balance elasticities that change to new equilibrium values only after the passage of time. In both mechanical and market systems this temporal lag in response to an applied force is a manifestation of friction.

Our identification of administered-rate dynamics as relaxations also provides a way of quantifying market friction. An expression for this dissipation—also known as ''internal friction'' in the anelasticity literature—can be obtained by considering the case of a periodic market scalar stress $\sigma^{(m)}(t)$ due to a periodic market force

$$\sigma^{(m)}(t)=\sigma^{(m)}(0)e^{i\omega t}, \tag{4.1}$$

where $\sigma^{(m)}(0)$ is the market stress at time $t=0$, $i=\sqrt{-1}$, and ω is the cyclic frequency of the market stress. The product rate will track the market force (and market rate, due to linearity as discussed above) with a lag that can be represented by a loss angle ϕ:

$$r^{(p)}(t)=r^{(p)}(0)e^{i(\omega t-\phi)}. \tag{4.2}$$

These expressions for the market and product rates imply a frequency dependent proportionality factor $J(\omega)$ [the Fourier transform of $J(t)$] that is complex $J(\omega)=J_1(\omega)-iJ_2(\omega)$ and a loss angle related to the components of $J(\omega)$ by $\tan(\phi)=J_2(\omega)/J_1(\omega)$.

The isomorphism between anelasticity and administered rates implies the existence of a state variable—a market equivalent of energy—at any phase in the market cycle given by $\int\sigma^{(m)}dr^{(p)}\propto\int r^{(m)}dr^{(p)}$ taken between the start of the cycle up to the point of interest. The market energy dissipated in a full market cycle is

$$\Delta U=\oint\sigma^{(m)}dr^{(p)}\propto\pi J_2[r^{(m)}(0)]^2, \tag{4.3}$$

and the stored market energy is

$$U=\int_{\omega t=0}^{\pi/2}\sigma^{(m)}dr^{(p)}\propto\frac{1}{2}J_1[r^{(m)}(0)]^2. \tag{4.4}$$

The ratio of these terms—the fractional market energy dissipated in a full market cycle—is related to the loss angle ϕ by $\Delta U/U=2\pi\tan(\phi)$ which, for the administered rates described by Eq. (2.4) yields

$$\tan\phi=\delta J\frac{\omega/\eta}{J_R+J_U\omega^2/\eta^2}. \tag{4.5}$$

Thus we see that the existence of an anelastic response ($\delta J\neq 0$) in a market system implies a dissipation of market energy and a formal definition of market friction. As the existence of this loss angle is due to the gathering and processing of information needed to reestablish equilibrium—$r^{(p)}\to\bar{r}^{(p)}$—it follows that the loss angle is also a measure of relative efficiency discussed by Farmer and Lo [22]. This is illustrated in Fig. 4 where we see the loss angle as a function of cyclic frequency ω for both the CMX and CD rates. In the limit $\omega\to 0$, the market cycle becomes so long that any finite relaxation rate is, on the time scale of a market cycle, instantaneous and the rate systems behave as if there is no relaxation with $J(t)=J_R$. In the limit $\omega\to\infty$, the market cycle becomes so short that the only component of the system that responds to the market force is the instantaneous portion J_U. Between these extremes we see the loss due to the relaxation behavior. For $\omega<0.16$ cycles per month (cycles greater than about 6 months) the CMX rate loss is greater than that of the CD. This is consistent with our expectation that the rate system that relaxes faster requires less market energy to achieve equilibrium and is, therefore, less lossy and more efficient. For $\omega>0.16$ (cycles less than about 6 months) the CD rate has a greater fractional loss per cycle. This perhaps unexpected result follows from the relative relaxation rates of these two systems. As the cyclic frequency increases, the relaxation component begins to ''freeze out'' and the system behaves in an increasingly elastic manner with $J(t)\to J_U$. Since the CD rate relaxes faster than the CMX rate, the relaxation component of the CMX rate ''freezes out'' first making it less lossy than the CD rate in this frequency range.

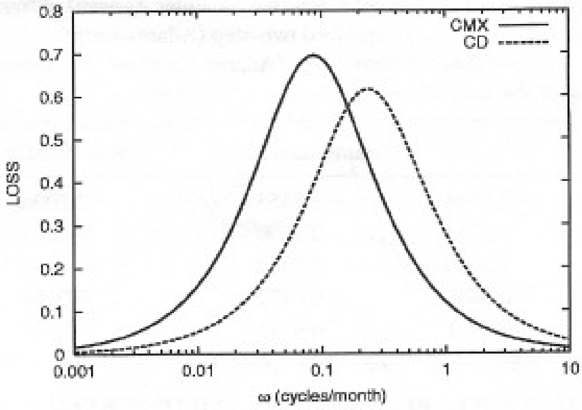

FIG. 4. The loss curves for CMX and CD rates.

V. DISCUSSION AND SUMMARY

Administered rates are unique in that they are set by a group of individuals attempting to maximize profits in the face of market forces. As the future direction of market forces, commonly measured by market rates, is unknown and committee decisions exhibit a degree of inertia, equilibrium between the market force and administered rate is achieved only after the passage of a certain amount of time. Historically this process has been formally expressed by an assumed linear equilibrium relationship and an *ad hoc* partial adjustment model to describe the change of the administered rate in response to a change in the market rate. Common to most previous treatments of administered-rate dynamics are the postulates that (i) for every market rate there is a unique equilibrium rate, and vice versa, (ii) the equilibrium response is achieved only after the passage of sufficient time, and (iii) the market-administered-rate relationship is linear. A contribution of this paper is the observation that these postulates also form the basis for a well-developed theory of relaxation processes in the physical sciences: indeed, these postulates are paraphrased directly from the introduction to anelasticity presented by Nowick and Berry [18]. We have examined this market system and found that the assumed equilibrium rate relationship corresponds to Hooke's law of elasticity and that the relaxation dynamics of administered rates are quite similar to anelastic relaxations. Developing this isomorphism we demonstrated that the basic structure of popular *ad hoc* partial adjustment models could be reproduced easily using standard techniques for discretizing the simplest anelastic differential relationship between the administered rate and market rate. We applied these models to the observed interest rate dynamics of a Cash Maximizer™ account and a certificate of deposit and found that, in spite of significant differences in the maturity features of these products, their dynamics are described well as anelastic relaxations. Finally we found that the anelastic description of these dynamics provides a natural definition of market friction as the realization of internal friction or dissipation in this market system.

ACKNOWLEDGMENTS

The authors thank Leif Wennerberg for criticisms of a previous draft that significantly improved this paper, David Anderson for help with the data used in this study, and Professor Daniel N. Beshers for enlightening conversations. Neither Bear, Stearns Securities Corporation or Deloitte & Touche are responsible for any statements or conclusions herein; and no opinions represented herein represent the position of Bear, Stearns Securities Corporation or of Deloitte & Touche.

[1] E. Mays, in *Controlling and Managing Interest-Rate Risk*, edited by A.G. Cornyn, J. Lederman, and R.A. Klein (New York Institute of Finance, New York, 1996), pp. 70–88.

[2] J. O'Brien, A. Orphanides, and S. Small, in *Proceedings of the Conference on Bank Structure and Competition* (Federal Reserve Bank of Chicago, Chicago, IL, 1994), pp. 400–435.

[3] A. Berger and T. Hannan, Rev. Econ. Stat. **71**, 291 (1989).

[4] L. Ausubel, manuscript, University of Maryland (unpublished).

[5] F. Diebold and S. Sharpe, J. Business Econ. Stud. **8**, 281 (1990).

[6] T. Hannan and A. Berger, Am. Econ. Rev. **81**, 938 (1991).

[7] T. Hannan and N. Liang, Finance and Economics Discussion Series No. 147, Board of Governors of the Federal Reserve System (unpublished).

[8] D. Newmark and S. Sharpe, Q. J. Econ. **107**, 657 (1992).

[9] D.E. Hutchison and G.G. Pennacchi, J. Financial Quant. Anal. **31**, 399 (1996).

[10] R.D. Selvaggio, in *Handbook of Asset/Liability Management*, edited by F.J. Fabozzi and A. Konishi (Irwin Professional, New York, 1996), Chap. 12, pp. 363–373.

[11] E.W. Irmler, *The OTS Net Portfolio Value Model* (Office of Thrift Supervision, Risk Management, Washington, DC, 1994).

[12] R.A. Jarrow and J.W. Levin, Am. Banker **March**, 18 (1996).

[13] J. O'Brien, working paper, Division of Research and Statistics, Board of Governors of the Federal Reserve System (unpublished).

[14] P. Kennedy, *A Guide to Econometrics*, 4th ed. (MIT Press, Cambridge, MA, 1998).

[15] S. Dattagupta, *Relaxation Phenomena in Condensed Matter Physics* (Academic Press, New York, 1987).

[16] R. Jarrow and D. van Deventer, J. Banking Finance **22**, 249 (1998).

[17] T. Janoshi, R. Jarrow, and F. Zullo, J. Derivatives **Fall**, 8 (1999).

[18] A.S. Nowick and B.S. Berry, *Anelastic Relaxations in Crystalline Solids* (Academic Press, New York, 1972).

[19] S.E. Koonin, *Computational Physics* (Benjamin Cummings, Menlo Park, CA, 1986).

[20] W.H. Press, S.A. Teukolsky, W. Vetterling, and B.P. Flannery, *Numerical Recipes in C: The Art of Scientific Computing*, 2nd ed. (Cambridge University Press, New York, 1994).

[21] Facts About Personal Deposit Account Programs, Bank of America, California, 1999.

[22] J. Farmer and A. Low, Santa Fe working paper No. 99-06-039, downloaded from http://www.santafe.edu/sfi/publications/99wplist.html (unpublished).